Single Correct Answer

1. **(b)** $u = 2\left(\frac{4}{2}+\frac{3}{1}\right) = 10$

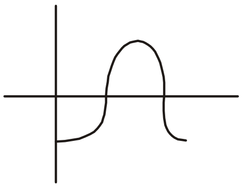
$\frac{1}{v}+\frac{1}{(-10)} = \frac{1}{+10}$; $v = +5$

$l_{app} = \left(\frac{3}{1}+\frac{9}{2}\right) = 3+4.5 = 7.5$

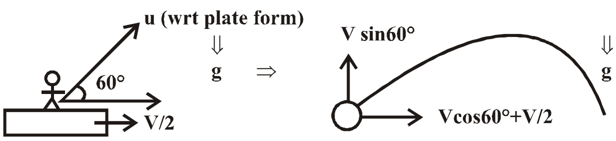
2. **(c)** $x = -A\sin\omega t$

$v = -A\omega\cos(\omega t)$

$a = +\omega^2 A\sin(\omega t)$



3. **(c)**

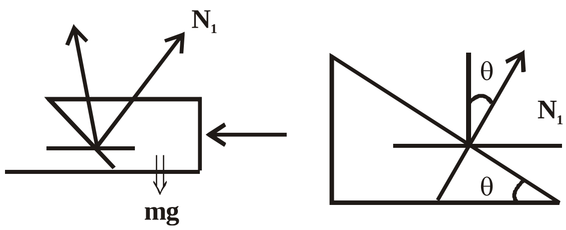


$R = \frac{2V\cos 60°.\,V\sin 60°}{g}$; $R' = \frac{2V.\,V\sin 60°}{g}$

$= \frac{2V^2\sin 60°.\cos 60°}{g}$; $R' = \frac{2V^2\sin 60°}{g}$

$\boxed{R' = 2R}$

4. **(c)**



$[N_1\sin\theta = F = N]$

5. **(a)**

6. **(c)**

$\lambda = \frac{h}{p} \Rightarrow \lambda - \frac{0.5}{100}\lambda = \frac{h}{p+\Delta p} \Rightarrow \frac{199\lambda}{200} = \frac{h}{p+\Delta p} = \frac{199}{200}\frac{h}{p}$

$\Rightarrow p+\Delta p = \frac{200}{199}p \Rightarrow p = 199\,\Delta p$

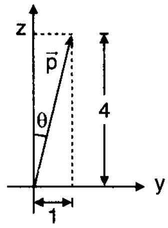
7. **(c)** $h = \frac{2T\cos\theta}{rdg}$ $\therefore \frac{h_2}{h_1} = \frac{T_2}{T_1}\times\frac{\cos\theta_2}{\cos\theta_1}\times\frac{d_1}{d_2}\times\frac{r_1}{r_2}$

$\frac{h_2}{h_1} = \frac{140}{70}\times\frac{\cos 60°}{\cos 0°}\times\frac{1}{2}\times 1 = \frac{1}{2} \Rightarrow h_2 = \frac{h_1}{2} = 3cm.$

8. **(a)** circuit in resonence so voltage accros resistance is equal to voltage applied, current is $i = \frac{V}{R}$

9.

q_i	x_i	$q_i x_i$	y_i	$q_i y_i$	z_i	$q_i z_i$
2q	0	0	a	2qa	a	2qa
q	0	0	–a	–qa	a	qa
–q	0	0	0	0	–a	qa



$p_x = \sum_i q_i x_i = 0$

$p_y = \sum_i q_i y_i = 2qa - qa$

$\Rightarrow p_y = qa$

$p_z = \sum_i q_i z_i = 2qa + qa$

$\Rightarrow p_z = 4qa$

Since, $\vec{p} = p_x\hat{i} + p_y\hat{j} + p_z\hat{k}$

$\Rightarrow \vec{p} = qa\,(\hat{j}+4\hat{k})$ (in yoz plane)

at an angle $\tan^{-1}\left(\frac{1}{4}\right)$ with the z-axis.

Passage -1 (10 to 12)

10. $\frac{1}{v}+\frac{1}{-15} = \frac{1}{+10} \Rightarrow \frac{1}{v} = \frac{15}{150}+\frac{10}{150} = \frac{25}{150}$

$v = \frac{150}{25} = 6$ cm.

11. $\bar{v}_{Im} = -\left(\frac{v}{4}\right)^2 \bar{v}_{om} \Rightarrow \bar{v}_{Im} = -\left(\frac{+6}{-15}\right)^2\times 3 = \frac{-4}{25}\times 3 = \frac{-12}{25}\hat{i}$

12. $h_I = h_o\left(-\frac{v}{u}\right)$ here v and u are constant

$(u = -10,\ v = +5)$

$\frac{dh_i}{dt} = \frac{dh_o}{dt}\left(-\frac{v}{u}\right) = \bar{v}_{Im} = -\left(\frac{+5}{-10}\right).\left(-3\hat{j}\right) = -1.5\hat{j}\ m/s$

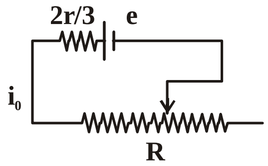
Passage - 2 (13 to 16)

13. (b), 14. (b), 15. (c), 16. (a)

$$E_{eq} = \frac{\left(\frac{2E}{r}-\frac{E}{2r}\right)}{\frac{1}{r}+\frac{1}{2r}} = \frac{\frac{3E}{2r}}{\frac{3}{2r}} = E$$

and $r_{eq} = \frac{2r}{3}$

So the arrangement will be

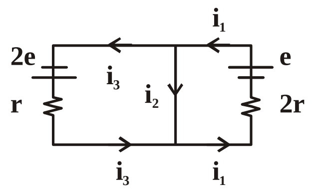


(i) Now power across R is maximum when internal resistance (r) = R i.e. $\boxed{R = 2r/3}$

(ii) $P_{max} = Ei = \frac{E^2}{\left(\frac{2r}{3}\right)} = \frac{3E^2}{2r}, i = i_{max.}$ (i is maximum when R = 0)

(iii) $i_0 = \frac{E}{\frac{2r}{3} + R}$; i_0 is maximum when R = 0 $\Rightarrow \boxed{i_0 = \frac{3E}{2r}}$

(iv)



$\left(i_1 = \frac{E}{2r}\right) \Rightarrow P_E = Ei_1 = \frac{E^2}{2r}.$

One or More than one Correct Answer

17. (b,c)

For adiabat 'bc'

$T_1V_b^{\gamma-1} = T_2V_c^{\gamma-1}$ ...(1)

For adiabat 'da'

$T_2V_d^{\gamma-1} = T_1V_a^{\gamma-1}$ ...(2)

Multiplying both (1) and (2)

$\Rightarrow T_1T_2(V_bV_d)^{\gamma-1} = T_1T_2(V_aV_c)^{\gamma-1}$

$\Rightarrow V_bV_d = V_aV_c$

Since adiabatic expansion leads to cooling.

so $T_1 > T_2$

18. (a, b, c) V = 2t

$L\frac{di}{dt} = 2t$

$2 \times \frac{di}{dt} = 2t \Rightarrow \frac{di}{dt} = t$

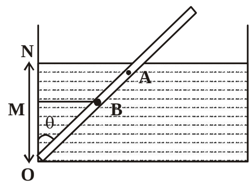
$i = \frac{t^2}{2} \Rightarrow i - t$ graph parabola

$U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 4 = 4J$

$\frac{dU}{dt} = Li\frac{di}{dt} = 2 \times \frac{t^2}{2} \times t = t^3$

$\frac{dU}{dt} = 1J/s$]

19. (a,b,d) Wt. of rod acts in downward direction, due to gravity Buoyant force in upward direction



$ON = 0.5 = \frac{1}{2}m$

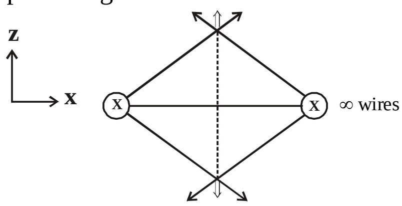
$OM = \frac{ON}{2} = \frac{1}{4}$

$\frac{OM}{OB} = \cos\theta$

$OB = \frac{OM}{\cos\theta} = \frac{1}{4} \times \frac{\sqrt{2}}{1} = \frac{1}{2\sqrt{2}}$

20. (a, b, c)

Draw x - z plane diagram



Along z-axis force on e^- will be towards origin hance stable equilibrium.

Matrix – Match Type

21. (A - r,s), (B - r,s), (C - r,s), (D - r,t)

For A, B and C– Magnetic field at location of 1 due to 2 is parallel or antiparallel to current in 1, so force experienced by 1 due to magnetic field of 2 is zero. For D–Force experienced by upper half of 2 due to 1 is along left, while on lower half it is towards right. So, net force of interaction between the two is zero.

Direction of magnetic field at P can be found by using RHPR No. 1.

Subjective Type

22. In the figures S $\rightarrow$ station. F $\rightarrow$ Factory and 'P' is the place where he meets the car.

usual day :

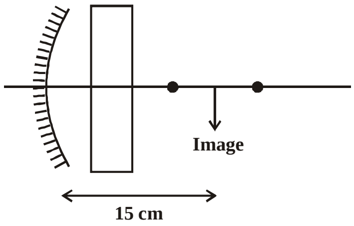
car starts from F at t = 0, reaches station at T and again reaches at the factory at time 2T.

This day :

Person reaches 'S' at T – 30. Car starts at t = 0 from F. Person walks for time t and reaches point P at time T – 30 + t. At this time car also reaches 'P'. Car comes back at 'F' at time (2T – 20). That means car takes time T – 10 from F to P. That means car reach at 'P' at time T – 5.

Now $T - 10 = T - 30 + t \Rightarrow t = 20$ min.

23.

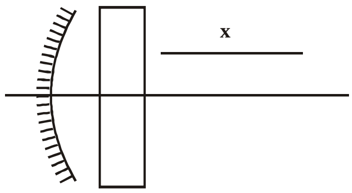


By the principle of reversibility. Let image be object than its image will be position of object.

u = 14 cm $\frac{1}{v} + \frac{1}{(-14)} = \frac{1}{(-10)}$

$$\frac{1}{v} = -\frac{1}{10} + \frac{1}{14} = \frac{-4}{140} \Rightarrow v = -\frac{75}{2}$$

Now mirror says to rays go and meet at $-\frac{75}{2}$



$$\left(\frac{x}{1} + \frac{3}{1.5} + \frac{2}{1}\right) = \frac{75}{2}$$

$$x = \frac{75}{2} - 4 = \frac{71}{2}$$

so distance will be $\left(\frac{71}{2} + 3 + 2\right)$ from mirror.

$= \frac{81}{2} = 40.5$ cm

so displacement is 10.5 towards right.

DETAILED SOLUTION

Single Choice Question

Sol 24.(d)

Mole ratio of C : H : O is 1 : 2 : 1 so empirical formula of CH_2O

$$m = \frac{\Delta T_b}{K_b} \Rightarrow \frac{0.15}{0.51} \Rightarrow 0.294;$$

$$0.294 = \frac{50}{M} \times \frac{1000}{950}; \; M \approx 180$$

$(CH_2O)_n = 180$ or $30 \times n = 180$ or $n = 6$;

$\therefore$ molecular formula is $C_6H_{12}O_6$.

Sol.25 (a)

Sol.26 (c) Fact.

Sol.27 (d) $\Delta H = (E_a)_f - (E_a)_b = 15 - 10 = 5$ kJ/mole

$E_{Tn} = 15 + 5 = 20$ kJ/mole

Sol. 28 **(c)** $P(V - b) = RT$

$PV = Pb + RT$

$y = mx + c$

(c) is correct answer.

Sol. 29 (a)

Sol. 30 (a)

Sol. 31. (c)

Sol.32 (d)

Sol.33 (c)

Sol. 34 (a)

Sol. 35 (c)

Sol. 36 (b)

Sol.37-(c); 38-(c); 39-(b)

One or More than One Correct Choice Type

Sol. 40. (a, c)

Sol.41. (b,d)

Sol.42 (b,d)

Sol.43 (a,c) $t_{99.99\%} = \frac{2.303}{k}\log\left(\frac{100}{0.01}\right) = \frac{4 \times 2.303}{k}$

$$t_{99\%} = \frac{2.303}{k}\log\left(\frac{100}{1}\right) = 2 \times \frac{2.303}{k}$$

Column Matching

Sol.44 A-r,s; B-p,q,r,s; C-p,q,r,s; D-p,q,r,s

Subjective Type

Sol.45 (4)

Sol. 46. (1)

$$\underset{\text{(cold, dil.)}}{Cl_2 + 6NaOH} \longrightarrow \underset{(X)}{NaCl} + \underset{(Y)}{NaClO} + H_2O$$

$$\underset{\text{(hot)}}{3Cl_2 + 6NaOH} \longrightarrow \underset{(X)}{5NaCl} + \underset{(Z)}{NaClO_3} + 3H_2O$$

In *Y*, *i.e.*, NaClO, the cation is Na^+,

$\therefore$ The oxidation state of Na is +1.

DETAILED SOLUTION

(SECTION - I, Single Correct Choice Type)

Sol.47(d) $\int_0^3 \left(-x^2 + ax + 12\right) dx = 45$

gives a = 4

Hence $f(x) = 12 + 4x - x^2 = (2 + x)(6 - x)$

hence m = – 2 and n = 6

m + n + a = 6 – 2 + 4 = 8.

Sol.48(a) By expanding about R_1

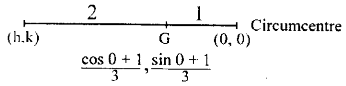
$(\tan^{-1} x)^3 + (\tan^{-1} 2x)^3 + (\tan^{-1} 3x)^3 = 3 \tan^{-1} x \tan^{-1} 2x \tan^{-1} 3x$

$\Rightarrow x = 0.$

Sol.49(b) $\boxed{A_1A_2}\ \boxed{B_1B_2}$ .... $\boxed{L_1L_2}$

number of ways in a circle $(11)!\ 2^{12}$.

Sol.50(a) Let C(cosθ, sinθ); H(h, k) is the orthocentre of the ΔABC

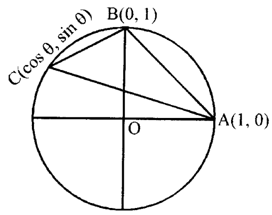


$h = 1 + \cos\theta$

$k = 1 + \sin\theta$

$(x - 1)^2 + (y - 1)^2 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0.$



Sol.51(c) Given $A^2 = A$

$I = (I - 0.4A)(I - \alpha A)$

$= I - I\alpha A - 0.4AI + 0.4\alpha A^2$

$= I - A\alpha - 0.4A + 0.4\alpha A$

$= I - A(0.4 + \alpha) + 0.4\alpha A$

hence $0.4\alpha = 0.4 + \alpha \Rightarrow \alpha = -2/3$.

Sol.52(c) $I = \int_0^{\pi/2} \frac{d}{dx}\left((\sin x)^x\right)dx = (\sin x)^x \Big|_0^{\pi/2}$

$= 1 - \lim_{x \to 0}(\sin x)^x = 1 - 1 = 0.$

Sol.53(a) A number is divisible by four, if the last two digit are divisible by four. In this case last two digits can be 12, 16, 28, 32, 36, 68, 92 or 96.

Total number of such numbers $= 8\,({}^4C_3 \cdot 3!) = 192$.

Sol.54(a) $f(x) = 2x^3 - 3x^2 + 6$

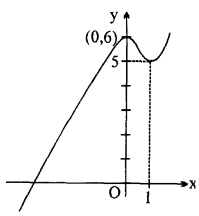
$f'(x) = 6x^2 - 6x = 6(x^2 - x) = 0$

gives x = 0 or x = 1

for inverse to exist function must be one one onto

hence Domain is $[1, \infty)$

Hence $a \geq 1$.



Comprehension Type (Paragraph for question Nos. 55 to 57)

Sol. (55 to 57) $\det.A = \begin{vmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{vmatrix}$

$= 2(-16 + 15) + 1(12 - 15) + 1(-15 + 20)$

$= -2 - 3 + 5 = 0 \qquad \Rightarrow$ A is singular

$\det.B = \begin{vmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{vmatrix} = 1(-12 + 12) + 1(12 - 12) + 1(-12 + 12)$

$= 0 \qquad \Rightarrow$ B is also singular

$\det.C = \begin{vmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{vmatrix} = -1(16 - 12) - 1(-12 + 9)$

$= -4 + 3 = -1 \qquad \Rightarrow$ C is non singular.

again $A^2 = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} = A$

$\Rightarrow$ A is idempotent

$B^2 = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}\begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow$ B is nilpotent

$C^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\Rightarrow$ C is involutary

55. (b) obvious (B) as (B) is nilpotent.

56. (c) $P = A^3C^2 = A^3 = A \qquad \Rightarrow P^2 = A^2 = A$

$\therefore P^2 = P$

|||1y in B and D

hence A^2B is not Idemnotent.

57. (d) Let $X = BC^2 \quad \Rightarrow \quad \det. X = 0$

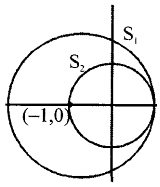
$Y = A^2C^2 \quad \Rightarrow \quad \det. Y = 0$

$Z = A^2B \quad \Rightarrow \quad \det. Z = 0$

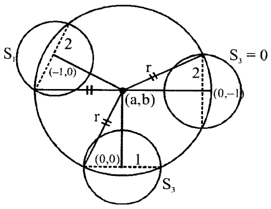
but $W = C^3 \quad \Rightarrow \quad \det. W \neq 0$

hence C^3 has an inverse $\Rightarrow$ (D)

Sol. (58 to 60)



58. (c)



$r^2 = a^2 + b^2 + 1 = (a+1)^2 + b^2 + 4$

$\Rightarrow$ 2a + 4 = 0 $\Rightarrow$ a = –2

and $(a+1)^2 + b^2 + 4 = a^2 + (b+1)^2 + 4$

$\Rightarrow$ 2a = 2b $\Rightarrow$ b = –2

$r^2 = 9 \Rightarrow r = 3$

59. (d) $S_1 - S_2 = 0 \quad \Rightarrow x = 1$

$S_2 - S_3 \quad \Rightarrow y = 1$

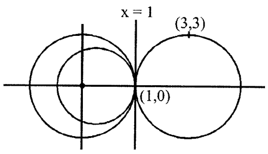
$\therefore$ Radical centre = (1, 1)

radius $L_T = \sqrt{S_1} = 1$

$\therefore$ equation of circle is $(x-1)^2 + (y-1)^2 = 1$

$\Rightarrow$ radius = 1 and a = 1 ; b = 1 $\Rightarrow$ a + b + r = 3

60. (c)



Family of circles touches the line x – 1 = 0 at (1, 0) is

$(x-1)^2 + (y-0)^2 + \lambda(x-1) = 0$

passing through (3, 2) $\Rightarrow$ 4 + 4 + 2λ = 0 $\Rightarrow$ $\lambda = -4$

$\therefore\ x^2 + y^2 - 6x + 5 = 0 \qquad \therefore$ radius $\sqrt{9-5} = 2$

Assertion & Reason Type

Sol.61 (a)

Sol.62 (c) $f(x) = \begin{bmatrix} \sin x & \text{if} & x > 0 \\ \dfrac{\tan \pi x^2}{x^2} & \text{if} & x < 0 \end{bmatrix}$;

Hence f (0⁺) = 0 as f (0⁻) = π]

SECTION - II (Multiple Correct Choice Type)

Sol.63 (b,c) $C_1(2,-3)$; $r_1 = \sqrt{4+9-8} = \sqrt{5}$

$C_2(5,3)$; $r_2 = \sqrt{25+9-14} = 2\sqrt{5}$

$C_1C_2 = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$\therefore\ C_1C_2 = r_1 + r_2$

Hence circle touch each other externally $\Rightarrow$ B

Hence radical axis is $S_1 - S_2 = 0$

i.e. x + 2y – 1 = 0 is also one of the common tangent $\Rightarrow$ C

(A) and (D) are obviously not correct.

Sol.64 (a,b,c,d)

Sol.65 (a,b,c,d)

$y = x^{1/3}(x-1)$

$\dfrac{dy}{dx} = \dfrac{4}{3}x^{1/3} - \dfrac{1}{3}\cdot\dfrac{1}{x^{2/3}} = \dfrac{1}{3x^{2/3}}[4x-1]$

hence f is ↑ for $x > \dfrac{1}{4}$

and f is ↓ for $x < \dfrac{1}{4}$

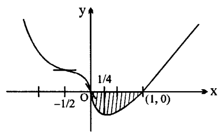
($x^{2/3}$ is always positive and x = 1/4 the curves has a local minima)

now $f'(x) = \dfrac{4}{3}x^{1/3} - \dfrac{1}{3}x^{-2/3}$

(non existent at x = 0 vertical tangent)

$f''(x) = \dfrac{4}{9}\cdot\dfrac{1}{x^{2/3}} + \dfrac{1}{3}\cdot\dfrac{2}{3}\cdot\dfrac{1}{x^{5/3}}$

$= \dfrac{2}{9x^{2/3}}\left[2 + \dfrac{1}{x}\right] = \dfrac{2}{9x^{2/3}}\left[\dfrac{2x+1}{x}\right]$



$\therefore\ f''(x) = 0$ at $x = -\dfrac{1}{2}$ (inflection point)

graph of f (x) is as

$A = \int_0^1 \left(x^{4/3} - x^{1/3}\right)dx = \dfrac{3}{7}x^{3/7} = -\dfrac{3}{4}x^{4/3}\Big]_0^1$

$= \left|\dfrac{3}{7} - \dfrac{3}{4}\right| = 3\left|\dfrac{4-7}{28}\right| = \dfrac{9}{28}$

Sol.66 (a,d)

$$S = 0\cdot\left({}^{100}C_0\right)^2 + 1\cdot\left({}^{100}C_1\right)^2 + ... + 100\cdot\left({}^{100}C_{100}\right)^2$$

$$S = 100\cdot\left({}^{100}C_0\right)^2 + 99\cdot\left({}^{100}C_1\right)^2 + ... + 0\cdot\left({}^{100}C_{100}\right)^2$$

$$2S = 100[({}^{100}C_0)^2 + \left({}^{100}C_1\right)^2 + ... + \left({}^{100}C_{100}\right)^2]$$

$$2S = 100.{}^{200}C_{100} = 100\cdot\frac{(200)!}{100!\cdot 100!}$$

$$\Rightarrow 2S = \frac{100.200.(199)!}{100.(99)!.(100)!} \Rightarrow S = 100.{}^{199}C_{99} \Rightarrow D$$

$$100.{}^{199}C_{99} = 100.\frac{199!}{99!100!} = \frac{100.(200)!}{200.99!100!}$$

$$= \frac{100.2^{100}.100!(1.35....199)}{200.99!.100!}$$

$$= \frac{2^{100}!.(1.3.5.....199)}{2.99!} = \frac{2^{99}(1.3.5.....199)}{99!} \Rightarrow A$$

SECTION - III (Matrix – Match Type)

Sol.67 [A - s ; B - p ; C - r ; D - q, r]

A. **Simple**

B. cos 40° – 2 sin 10° . cos 40°

cos 40° – [sin 50° – sin 30°]

$\cos 40° - \sin 50° + \frac{1}{2}$

C. **Simple**

SECTION - IV (Subjective Type)

Sol.68 [0272]

$x_1 = 4$ **or** $x_2 = 16$ **;** $x_1^2 + x_2^2 = 272$

$$3^{\log_2 x} - 12\cdot x^{\log_{16} 9} = \log_3\left(\frac{1}{3}\right)^{3^3}$$

$$\text{or } x^{\log_2 3} - 12\cdot x^{\log_4 3} = \log_3\left(\frac{1}{3^{3^3}}\right)$$

$$\text{or } x^{\log_2 3} - 12\cdot x^{\frac{1}{2}\log_2 3} = -\log_3 3^{3^3} = -27$$

Let $x^{\log_2 3} = y$

$y - 12\sqrt{y} + 27 = 0$

$\therefore \left(\sqrt{y} - 9\right)\left(\sqrt{y} - 3\right) = 0$

$\therefore$ y = 81 or y = 9

hence $x^{\log_2 3} = 3^4$ or $3^{\log_2 x} = 3^4 \Rightarrow x = 16$

or $3^{\log_2 x} = 3^2 = 3^2 \Rightarrow x = 4$

So, $x_1^2 + x_2^2 = 272$

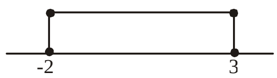
Sol.69 [0004]

Since a > 0 $\therefore f(2) < 0$

$\therefore 4 - 2(k+1) + k^2 + k - 8 < 0$

$k^2 - k + 6 < 0$

$(k+2)(k-3) < 0$



$\therefore k \in (-2, 3)$

Integral value of k is –1, 0, 1, 2 ⇒ number of Integral value of k is 4